MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

PEM0036 – CALCULUS

(Foundation in Engineering – All sections / Groups)

6 MARCH 2017
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 6 pages.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.

4. Only **NON-PROGRAMMABLE** calculator is allowed.

APPENDIX

BASIC DIFFERENTIATION AND INTEGRATION FORMULAS

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[e^x] = e^x \qquad \frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$$

$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}; \quad |x| < 1 \qquad \frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}; \quad |x| < 1$$

$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2-1}}; \quad |x| > 1 \qquad \frac{d}{dx}[\csc^{-1} x] = -\frac{1}{|x|\sqrt{x^2-1}}; \quad |x| > 1$$

$$Area = \int_a^b [f(x) - g(x)]\ dx$$

$$Volume\ (Disk) = \pi \int_a^b [f(x)]^2\ dx$$

$$Volume\ (Washer) = \pi \int_a^b [f(x)]^2 - [g(x)]^2\ dx$$

$$Volume\ (Cylindrical\ Shells) = \int_a^b 2\pi (shell\ radius)(shell\ height)\ dx$$

Continued...

QUESTION 1 [25 marks]

a. Given $f(x)=\begin{cases} 2x+4 & ; \quad x\le -3 \\ \dfrac{x+3}{x^2-9} & ; \quad -3<x<3 \\ 10 & ; \quad x\ge 3 \end{cases}$,

Find

i. $\lim\limits_{x\to 4} f(x)$ [1 mark]

ii. $\lim\limits_{x\to -3^+} f(x)$ [3 marks]

Show that

iii. $\lim\limits_{x\to -3} f(x)$ does not exist. [4 marks]

iv. the function $f(x)$ is continuous at $x=-4$. [2 marks]

b. Evaluate the following limits:

i. $\lim\limits_{x\to 0} 2\left(\dfrac{3+2e^{-2x}}{x-1}\right)$. [2 marks]

ii. $\lim\limits_{x\to -1} \dfrac{\sqrt{x^2+35}-6}{x+1}$. [4 marks]

iii. $\lim\limits_{x\to 3^+} \dfrac{|3-x|}{3-x}$. [2 marks]

c. Given $f(x)=\left(\dfrac{4-2x-x^2}{2x^2-8}\right)$, determine the $\lim\limits_{x\to\infty} f(x)$ and the asymptote(s) of $f(x)$.

[7 marks]

Continued...

QUESTION 2 [25 marks]

A certain curve is expressed as $f(x)=\frac{10x}{2x^2+7}$. Do the following:

(Use exact value throughout the computation)

a. Find the absolute minimum and maximum coordinate in the interval $[-5,5]$.
[11 marks]

b. Use first derivative method to determine the local extreme value(s) and where they occur in the $f(x)$ domain. [3 marks]

c. If $f''(x)=\frac{40x(2x^2-21)}{(2x^2+7)^3}$, use the given second derivative to verify your answer in part (b). [5 marks]

d. The values of $x=0$ and $x=\pm\sqrt{\frac{21}{2}}$ are obtained when $f''(x)=0$ or undefined. Determine the concavity interval. [6 marks]

Continued...

QUESTION 3 [25 marks]

a. Find each of the following:

i. Given that $\frac{d}{dx}\left[-\frac{1}{2}e^x(\cos x-\sin x)\right]=e^x\sin x$, find $\int\left(e^x\sin x\right)\,dx$. [1 mark]

ii. $\int\left[8\left(\sqrt[5]{x^3}\right)+2e^x+3\pi\right]\,dx$ [3 marks]

iii. $\int\frac{(x-2)(x+4)}{(x+1)(x-2)^3}\,dx$ [8 marks]

b. Evaluate $\int_0^{\frac{\pi}{2}}\left(\sin^5 x\right)dx$. [6 marks]

$\left[\text{Hint}: \sin^2 x=1-\cos^2 x\right]$

c. i. Find the volumes of the solid generated when the shaded region R in the Figure Q3 is rotated about the y- axis. [5 marks]

ii. Set up, but do not integrate an integral expression in terms of a single variable for the volumes of the solid generated when the shaded region R is revolved about the x- axis. [2 marks]

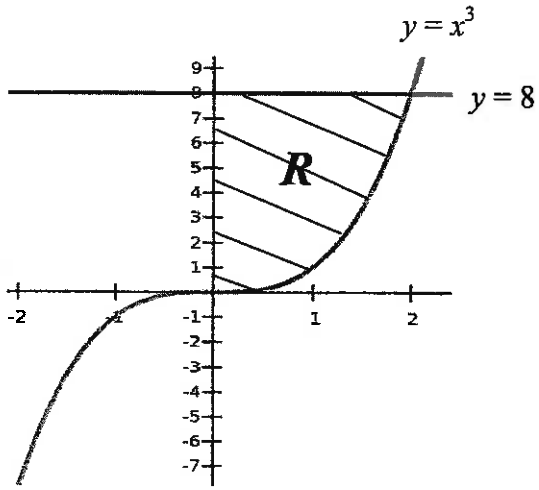

Figure Q3

Continued...

QUESTION 4 [25 marks]

a. Find the function that satisfies the following initial value problem:

$3y' = 6xy \quad ; \quad y(0) = 4.$ [5 marks]

b. Solve the given first order differential equation: $\frac{dy}{dx} = e^{4x} + 2y.$ [5 marks]

c. Solve the second order differential equation:
$\left(e^{-3x}\right)y'' + \left(e^{-3x}\right)(6y' + 9y) = 72$, given $y(0) = 3$ and $y'(0) = 6.$ [15 marks]

End of Page